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Linear programming and model predictive control

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Abstract

The practicality of model predictive control (MPC) is partially limited by the ability to solve optimization problems in real time. This requirement limits the viability of MPC as a control strategy for large scale processes. One strategy for improving the computational performance is to formulate MPC using a linear program. While the linear programming formulation seems appealing from a numerical standpoint, the controller does not necessarily yield good closed-loop performance. In this work, we explore MPC with an l_1 performance criterion. We demonstrate how the non-smoothness of the objective function may yield either dead-beat or idle control performance.

Keywords: Model predictive control; Linear programming; Optimization

1. Introduction

Model predictive control (MPC) is an optimization based strategy that uses a plant model to predict the effect of potential control action on the evolving state of the plant. At each time step, an open-loop optimal control problem is solved and the input profile is injected into the plant until a new measurement becomes available. The updated plant information is used to formulate and solve a new open-loop optimal control problem.

The MPC methodology is appealing to the practitioner, because input and state constraints are explicitly accounted for in the controller. A practical disadvantage is the computational cost, which tends to limit MPC applications to linear processes with relatively slow dynamics. For such problems, the optimal control problem to be solved at each stage of MPC is a convex program. The necessity to solve the optimization problem in real time is especially troublesome for large-scale processes. While efficient software exists for the solution of convex programs, significant improvements are obtained by exploiting the structure of the MPC subproblem.

Traditionally model predictive control has been formulated using a quadratic criterion. Part of the popularity of the quadratic criterion from a theoretical standpoint is due to its mathematical convenience. From a numerical standpoint, the quadratic criterion is popular, because the resulting optimization can be cast as a quadratic program. For the unconstrained case, the linear quadratic optimal control problem is solved efficiently using dynamic programming. This solution technique has the desirable property that the computational cost scales linearly in the horizon length N as opposed to cubically for the general least squares solution. While the addition of constraints negates the possibility of a general analytic solution to the optimal control problem, the quadratic program may be structured in an analogous manner to the unconstrained problem, yielding linear growth in the horizon length N. Approaches to structuring the optimal control problem with a linear quadratic objective utilizing sparse matrix methods are available in the literature [2,18,21].

Recently Dave and co-workers [7] have advocated the use of an l_1/l_∞ norm as a performance criterion for MPC. One motivation is that the resulting optimal control problem is cast as a linear program. The solution of a linear program is less computationally demanding than the corresponding solution of a quadratic program of the same size and complexity, so it may be preferable to formulate MPC as a linear program. The concept of using linear programming is not new and has been considered by many authors in optimal control (e.g. [16,22]) and in MPC (e.g. [1,3–6,10,12,14,17]). A review of some MPC research with non-quadratic objectives can be found in the paper by Garcia and co-workers [9]. The main theoretical objection to linear programming formulations is that analytic

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solutions are generally unavailable due to the non-smoothness of the objective function. The non-smoothness is one of the prime reasons why the analysis of the stability for linear programming formulations has been lacking. Notable exceptions include the works of Keerthi and Gilbert [13], who use an endpoint constraint, Genceli and Nikolaou [10], who consider finite impulse response models, and Shamma and Xiong [20], who provide a numerical test whether a given horizon is sufficiently long to guarantee stability for unconstrained MPC.

In this paper we examine linear programming formulations of MPC. We begin our discussion by presenting in Section 2 a stabilizing formulation of MPC with a general l_p criterion. In Section 3 we analyze the qualitative properties of MPC with an l_1 criterion. Unlike MPC with a quadratic criterion, the choice of the tuning parameters for the l_1 formulation may result in appreciably different closed-loop performance. In particular, we demonstrate how the non-smoothness of the objective may yield either dead-beat or idle control performance.

2. Stabilizing MPC with l_p criterion

Consider the regulation following linear discrete-time representation of the plant

$$x_{k+1} = Ax_k + Bu_k, \quad k \geqslant 0, \tag{1a}$$

$$y_k = Cx_k \tag{1b}$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^m$, and $y_k \in \mathbb{R}^q$. We formulate the regulator as the feedback law $\eta(\hat{x}_j)$ that generates the sequence $\{u_k\}_{k=0}^{\infty}$, where $\eta(\hat{x}_j) \triangleq u_0$, that minimizes the infinite horizon objective function

$$\Phi(\hat{x}_k) = \sum_{k=0}^{\infty} \| \overline{R} u_k \| \hat{p} + \| \overline{Q} y_k \|_p, \tag{2}$$

subject to Eq. 1(a and b), the initial condition $x_0 = \hat{x}_j$, and the constraints

$$u_{\min} \leqslant Du_k \leqslant u_{\max}, \tag{3a}$$

$$-\Delta_u \leqslant \Delta u_k \leqslant \Delta_u, \tag{3b}$$

$$y_{\min} \leqslant y_k \leqslant y_{\max}, \tag{3c}$$

where

$$\| x \|_p \triangleq \left(\sum_{i=1}^{n} |x^{(i)}|^p \right)^{1/p}$$

and $x^{(i)}$ denotes the ith entry of the vector x. Common examples of l_p norms are the sum norm (l_1 norm)

$$\| x \|_1 \triangleq |x^{(1)}| + \cdots + |x(n)|$$

and the max norm (l_∞ norm)

$$\| x \|_\infty \triangleq \max\left\{ |x^{(1)}|, \cdots, |x^{(n)}| \right\}.$$

The vector $\hat{x}_j$ denotes the current state estimate of the plant at time index j. By suitably adjusting the origin, the regulator can account for target tracking and disturbance rejection [15]. We make the following assumptions: (a) (A, B) is stabilizable and (C, A) is detectable; (b) $\overline{Q}$ and $\overline{R}$ are diagonal matrices with positive elements; and (c) the origin $(u_k, x_k) = 0$ is contained within the interior of the feasible region Eq. 3(a–c). If a feasible solution exists, then the origin is an asymptotically stable fixed point for the feedback controller [13].

With the notable exceptions discussed in Section 3, analytic solutions to Eq. (2) are generally unavailable, because the l_p norm has a kink at the origin (see Fig. 1). To circumvent the computational barrier imposed by the infinite horizon calculation, we employ a stable finite horizon approximation. Our method is analogous to the technique employed by Rawlings and Muske [19] for a quadratic criterion. The basic strategy is to consider only a finite number of decision variables, so that the infinite horizon problem reduces to a finite dimensional mathematical program.

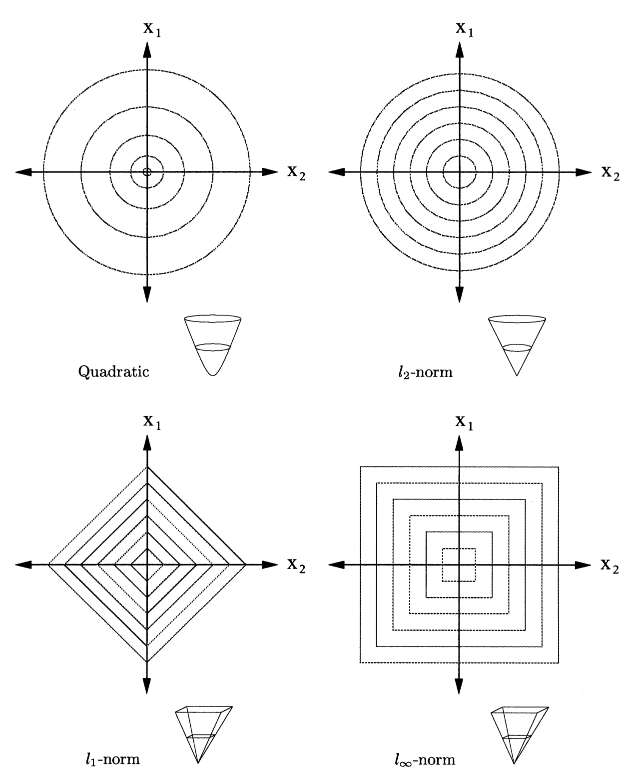

Fig. 1. Geometric interpretation of cost functions.

We transform the infinite horizon problem to a finite horizon problem with a terminal state penalty by considering the free evolution of only the stable modes on the infinite horizon. We obtain the transformation using the following terminal penalty

$$V(x) = \sum_{k=0}^{\infty} \left\| \overline{Q} C A_s^k x \right\|_p,$$

where A_s is the restriction of A to the stable subspace of A. For the majority of systems an analytic expression for $V(x)$ is unavailable. One simple strategy to generate a stable approximation for $V(x)$ is to assume that the non-zero eigenvalues of A_s are nondefective. This assumption allows us to upper bound the sum with a Lyapunov function. Consider the Jordan decomposition

$$A_s = S \begin{bmatrix} \Lambda & 0 \\ 0 & J_{n_0}(0) \end{bmatrix} S^{-1},$$

where the diagonal matrix Λ contains the non-zero eigenvalues of A_s and n_0 is the algebraic multiplicity of the zero eigenvalue. Because the Jordan block $J_{n_0}(0)$ is nilpotent, we have

$$A_s^n = [S_\Lambda S_0] \begin{bmatrix} \Lambda^n & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \left(S^{-1}\right)_\Lambda \\ \left(S^{-1}\right)_0 \end{bmatrix}, \tag{4a}$$

$$= S_\Lambda \Lambda^n \left(S^{-1}\right)_\Lambda, \tag{4b}$$

for all $n \geqslant n_0$. If we consider the coordinate transformation $z = \left(S^{-1}\right)_\Lambda x$, we generate the following upper bound

$$\overline{V}(x) = \sum_{k=0}^{n_0-1} \left\| \overline{Q} C A^k x \right\|_p + \theta^T |z|,$$

where $|z|$ is a vector whose entries are the absolute value of the associated entries of z and

$$\theta^{(j)} = \sum_{k=n_0}^{\infty} \left\| \overline{Q} C S_\Lambda \Lambda^k e_j \right\|_p,$$

where the vector e_j is the unit vector whose jth entry is 1. The validity of this bound follows directly from the subadditivity of norms:

$$\sum_{k=n_0}^{\infty} \left\| \overline{Q} C A_s^k x \right\|_p =$$

$$\sum_{k=n_0}^{\infty} \left\| \overline{Q} C S_\Lambda \Lambda^k_{(S^{-1})} \Lambda x \right\|_p =$$

$$\sum_{k=n_0}^{\infty} \left\| \overline{Q} C S_\Lambda \Lambda^k \left(z^{(1)} e_1 + \cdots + z^{(n-n_0)} e_{n-n_0} \right) \right\|_p$$

$$\leq \sum_{k=n_0}^{\infty} \sum_{j=1}^{n-n_0} \left| z^{(j)} \right| \left\| \overline{Q} C S_\Lambda \Lambda^k e_j \right\|_p = \theta^T |z|.$$

Lemma 2.1. $\overline{V}(x) \geqslant \| QCx \|_p + \overline{V}(A_s x)$.

Proof. It follows from Eq. (4a) that $\left(S^{-1}\right)_\Lambda A_s x = \Lambda z$. Hence, we have

$$\overline{V}(A_s x) - \overline{V}(x) = \left\| QCS_\Lambda \Lambda^{n_0} z \right\|_p + \theta^T |\Lambda z| - \left(\left\| QCx \right\|_p + \theta^T |z| \right).$$

Expanding $\theta^T z$, we generate the follow inequality.

$$\begin{aligned} \theta^T |x| &= \sum_{j=1}^{n-n_0} \left| z^j \right| \overline{Q} C S_\Lambda \Lambda_0^n e_j \|_p + \\ & \left| z^j \right| \sum_{k=n_0}^{\infty} \| \overline{Q} C S_\Lambda \Lambda^k \Lambda e_j \|_p, \\ &\geqslant \| \overline{Q} C S_\Lambda \Lambda^{n_0} z \|_p + \\ & \sum_{k=n_0}^{\infty} \sum_{j=1}^{n-n_0} \left| \Lambda^{(j)} z^{(j)} \right| \| \overline{Q} C S_\Lambda \Lambda^k e_j \|_p, \\ &= \| \overline{Q} C S_\Lambda \Lambda^{n_0} z \|_p + \theta^T |\Lambda z|. \end{aligned}$$

Hence the lemma follows.

We formulate the finite horizon regulator as the solution to

$$\Phi_N^*(\hat{x}_k) = \min_{u_k, x_k} \sum_{k=0}^{N-1} \left\| \overline{R} u_k \right\|_p + \left\| \overline{Q} y_k \right\|_p + \overline{V}(x_N), \tag{5}$$

subject to Eq. 1(a and b), the initial condition $x_0 = \hat{x}_j$, Eq. 3(a–c), and

$$F^T x_N = 0 \tag{6}$$

where the columns of F span the orthogonal complement of the stable subspace of A. An ordered Schur decomposition of A yields an orthogonal representation of F. In the absence of the constraints Eq. 3(a–c), choosing $N \geqslant n$ is sufficient to guarantee feasibility. With the presence of inequality constraints, feasibility is obtained for stable systems if N is sufficiently large such that

$$x_N \in \mathcal{O}_\infty \tag{7}$$

where the set $\mathcal{O}_\infty$ is positive invariant and contained within the feasible region specified by Eq. 3(a–c). Details concerning the properties and construction of O_∞ are available in the work of Gilbert and Tan [11]. For unstable systems, we also require that the state $\hat{x}_j$ is contained in the set of constrained stabilizable states and N is sufficiently large such that Eq. (6) is feasible.

Proposition 2.2. *If a feasible solution exists, then the origin is an asymptotically stable fixed point for the closed-loop system.*

Proof. Stability follows from the continuity of Φ_N^*. To demonstrate convergence, let

$$\{u_{k|k}, \cdots, u_{k+N-1|k}\}$$

denote the minimizing sequence at time index k. Since the sequence

$$\{u_{k+1|k}, \cdots, u_{k+N|k}, 0\}$$

is also admissible at time $k+1$, we have from Lemma 2.1 that

$$\Phi_N^*(\hat{x}_k) - \Phi_N(\hat{x}_{k+1|k}) \geqslant \left(\|\overline{R}u_{k|k}\|_p + \|\overline{Q}y_{k|k}\|_p\right).$$

The sequence $\{\Phi_N^*(x_k)\}_{k=0}^{\infty}$ is convergent, because it is nonincreasing and bounded below. Hence,

$$\left(\|\overline{R}u_{k|k}\|_p + \|\overline{Q}y_{k|k}\|_p\right) \rightarrow 0$$

as $k \rightarrow \infty$. Because (C, A) is detectable, we have $x_k \rightarrow 0$. Therefore, the regulator is asymptotically stable as claimed.

2.1. Linear programming formulations

With either an l_1 or l_∞ criterion, we may transform the optimal control problem to a linear program by introducing auxiliary variables. We formulate (5) with an l_1 criterion as the following linear program

$$\Phi_N(\hat{x}_j) = \min_{x_k, u_k, \rho_k, \eta_k, \gamma_k, z_N}$$

$$\sum_{k=0}^{n-1} \mathrm{e}^T\rho_k + \mathrm{e}^T\eta_k + \sum_{k=0}^{n_0-1} \mathrm{e}^T\gamma_k + \theta^T z_N,$$

subject to Eq. 1(a and b) the initial condition $x_0 = \hat{x}_j$, Eqs. 3(a–c) and (6), where the non-negative vectors ρ_k, η_k, γ_k, and z_N are specified by the following linear inequalities

$$-\rho_k \leqslant \overline{R}u_k \leqslant \rho_k, \qquad -\eta_k \leqslant \overline{Q}Cx_k \leqslant \eta_k,$$

$$\gamma_k \leqslant \overline{Q}CA_s^k x_N \leqslant \gamma_k, -z_N \leqslant \left(S^{-1}\right)_\Lambda x_N \leqslant z_N.$$

With an l_∞ criterion, we formulate (5) as the following linear program

$$\Phi_N(\hat{x}_j) = \min_{x_k, u_k, \rho_k, \eta_k, \gamma_k, z_N}$$

$$\sum_{k=0}^{\infty} \rho_k + \eta_k + \sum_{k=0}^{n_0-1} \gamma_k + \theta^T z_N,$$

subject to Eq. 1 (a and b), the initial condition $x_0 = \hat{z}_j$, Eqs. 3(a–c) and (6), where the non-negative scalars ρ_k, η_k, and γ_k and the vector z_N are specified by the following linear inequalities

$$-\rho_k e \leqslant \overline{R}u_k \leqslant \rho_k e, \quad -\eta_k e \leqslant \overline{Q}Cx_k \leqslant \eta_k e,$$

$$-\gamma_k e \leqslant \overline{Q}CA_s^k x_N \leqslant \gamma_k e,$$

$$-z_N \leqslant \left(S^{-1}\right)_\Lambda x_N \leqslant z_N.$$

The variable e is the vector of ones.

3. MPC with an l_1 norm objective

Consider the regulation of the following non-minimum phase system

$$y(s) = \frac{s-3}{3s^2+4s+2} u(s),$$

sampled at frequency of 10 Hz with an initial state disturbance of $x_0 = [1, 1]^T$. A horizon length of $N = 30$ was chosen for both examples. For simplicity we ignore inequality constraints, because they add little to the theme of the discussion on qualitative performance. Fig. 2 shows the comparison of the closed-loop responses between an l_1 criterion and a quadratic criterion with tuning parameters $\overline{Q} = 5$ and $\overline{R} = 1$. The simulation indicates the l_1 formulation forces the state to the origin in finite time as opposed to the quadratic programming formulation, where the state exponentially approaches the origin. Further simulations indicate that the dead-beat policy holds for all initial conditions. The finite horizon problem is also equivalent to the infinite horizon problem, because the l_1 formulation forces the state to the origin in finite time. Forcing the state to the origin in finite time is appealing for servo regulation. However, dead-beat control may yield poor closed-loop

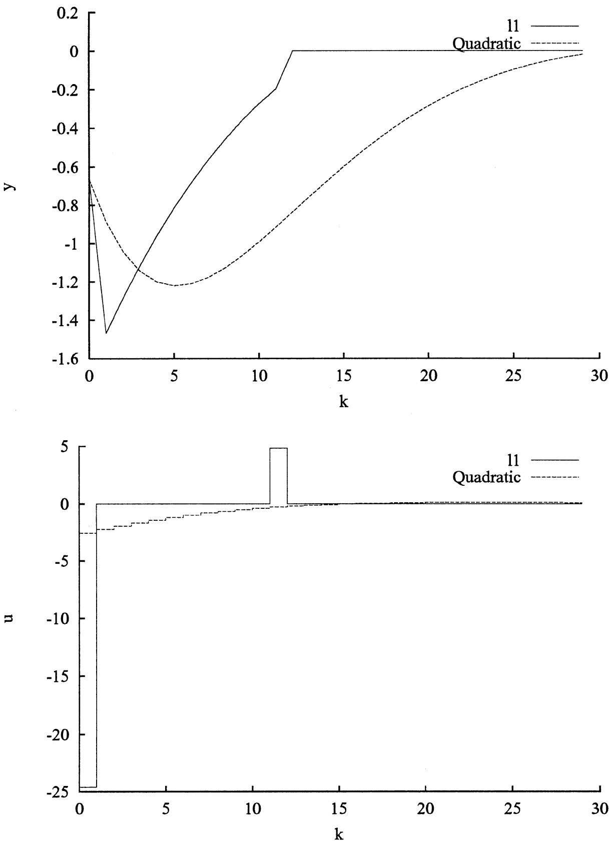

Fig. 2. Comparison of input and output responses for $Q = 5$ and $R = 1$.

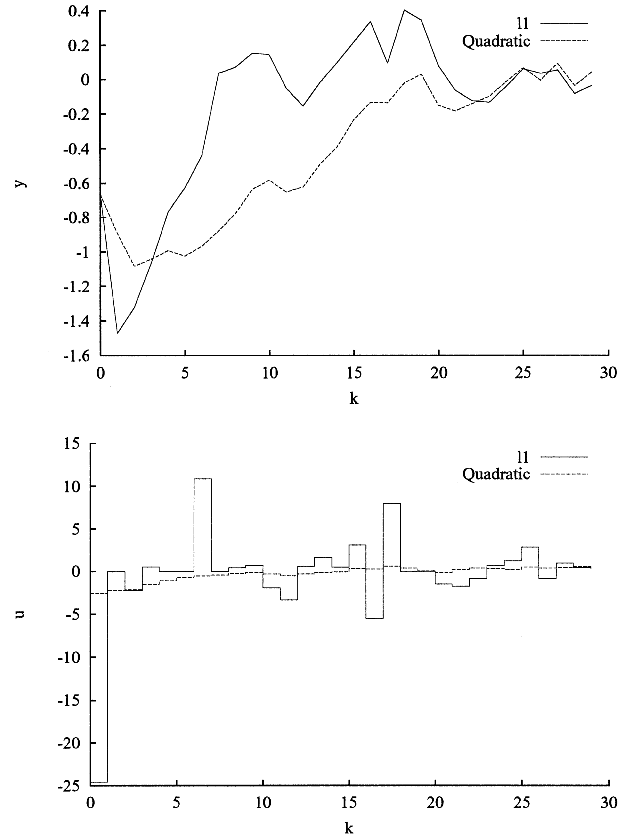

Fig. 3. Comparison of input and output responses for $Q = 5$ and $R = 1$ with state noise.

performance in process control applications. The poor performance becomes evident when state noise is added to the simulation. Fig. 3 shows a comparison of closed-loop responses when state noise is added. The deviation from the target is less for the l_1 formulation; at the same time the dead-beat performance causes aggressive control action. In many situations this high-gain control is undesirable.

In addition to yielding dead-beat performance, the l_1 formulation results in idle control performance when the input penalty $\overline{R}$ is large relative to $\overline{Q}$. Fig. 4 shows the comparison of the closed-loop responses between the l_1 criterion and the quadratic criterion with tuning parameters $\overline{Q} = 1$ and $\overline{R} = 5$. The simulation indicates that the optimal policy for the l_1 formulation is no control action. For the given tuning, the idle policy holds regardless of the initial conditions and the horizon length. Although the qualitative performance, e.g. the settling time, between the quadratic and l_1 criterion is not appreciably different for the example, idle control defeats the purpose of implementing a control system. The reason for the similarity between the open-loop response (idle control policy) and the closed-loop response (quadratic criterion) is that the large input penalty $\overline{R}$ relative to the output penalty $\overline{Q}$ pacifies the controller and, therefore, does not place the closed-loop poles of the system far from the open-loop poles. So, unlike the dead-beat policy, with the addition of disturbances, the qualitative performance of the idle policy will be similar to the performance of the quadratic formulation.

The two examples demonstrate that the l_1 formulation yields different qualitative performances depending on the selection of the tuning parameters. This dichotomy is in direct contrast to the quadratic formulation, where the qualitative control performance is similar regardless of the tuning; i.e. the qualitative performance is always exponential convergence. The difference between the two formulations is analogous to the difference between a positive definite quadratic program and a linear program. Whereas the solution to the quadratic program may reside in the interior of the feasible region, the solution to a linear program always resides at an extreme point of the feasible region.

We can specifically attribute the differences between the l_1 and quadratic formulation to the non-smoothness of the objective function. We can interpret the input and output stage costs as competing exact penalties, because the objective function for the l_1 criterion is a sum of norms (an introductory explanation of exact penalties may be found in Fletcher [8]). The purpose of exact penalties is to recast the constrained optimization

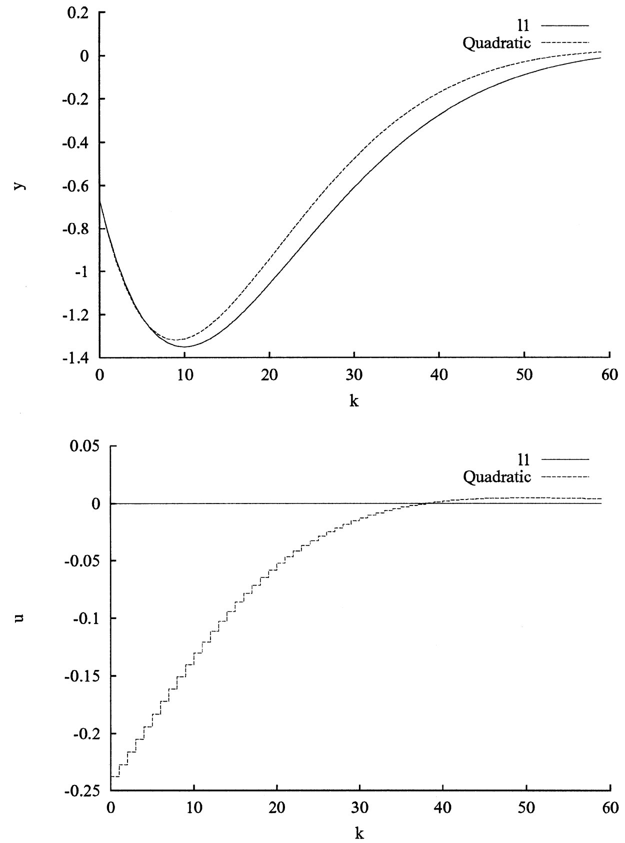

Fig. 4. Comparison of input and output responses for $Q = 5$ and $R = 5$

$$\min_x \left\{ f(x) : g(x) = 0 \right\}$$

as the equivalent unconstrained optimization

$$\min_x f(x) + \lambda \left\| g(x) \right\|_p.$$

If $\lambda > 0$ is sufficiently large (greater than the dual norm of the Lagrange multiplier associated with the constraint $g(x) = 0$), then the solutions to the two optimization are equivalent. Hence, we may view the terms $\left\| \overline{R} u_k \right\|_1$ as penalties for the constraint $u_k = 0$ and the terms $\left\| \overline{Q} y_k \right\|_1$ as penalties for the constraint $y_k = 0$. When the input penalty R is sufficiently large, the exact penalty $u_k = 0$ becomes a binding constraint. Likewise, when the output penalty $\overline{Q}$ is sufficiently large, the exact penalty $y_k = 0$ becomes a binding constraint. In particular, the two competing penalties are between dead-beat and idle control performance. We can also expect the same qualitative behavior for the l_∞ formulation, because the non-smoothness is present for any l_p formulation,

We demonstrate the effect of the non-smoothness of the objective function geometrically with a simple scalar example. Consider the following single stage optimal control problem

$$\min_{u_0} \Theta = |x_1| + r|u_0|,$$

subject to the scalar system

$$x_1 = ax_0 + bu_0.$$

Recognize that because both the state and input are scalar, this example encompasses all l_p norm formulations. Fig. 5 shows the graph of Θ as a function of u_0. It is evident from the graph that if $r > b$, then the optimal solution is $u_0 = 0$, because the slope of the middle section is negative. Likewise, if $r < b$, the optimal solution is $u_0 = -\dfrac{ax_0}{b}$, which yields dead-beat control, because the slope of the middle section is positive. If $r = b$, the optimal solution is not unique. Both solutions are optimal, including all solutions in between: either $0 \leqslant u_0 \leqslant -ax_0/b$ or $-ax_0/b \leqslant u_0 \leqslant 0$. If we consider the quadratic criterion

$$\Theta' = x_1^2 + ru_0^2$$

then the optimal solution is

$$u_0 = -\frac{ab}{b^2 + r} x_0.$$

In contrast to the l_1 formulation, the quadratic control is neither dead-beat nor idle for $r > 0$.

At this stage we are confronted with the question as to whether the l_p formulation is preferable to a quadratic formulation. In addition to the numerical advantages offered by linear programs, for many applications the actual control specifications translate more naturally into an l_p criterion than a quadratic criterion. However, the sensitivity of closed-loop behavior for the l_1 formulation is disconcerting, because the tuning parameters must be chosen judiciously to exclude undesirable

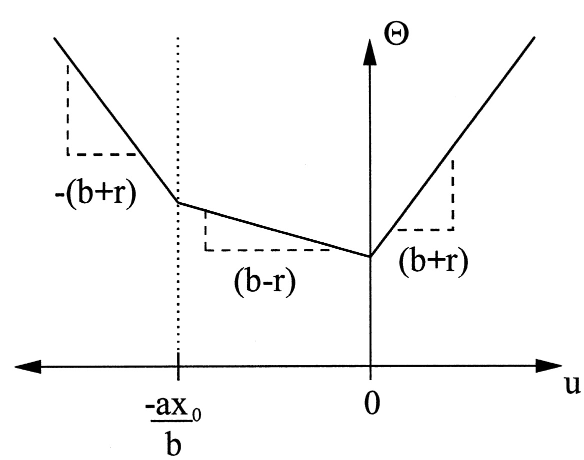

Fig. 5. Graph and slopes of the cost function Θ.

performance. Not only does one have to be wary of the implications of dead-beat or idle performance, non-uniqueness of the control presents potential problems, because erratic closed-loop behavior may result. We expect that additional measures such as input velocity penalties would help counteract the aggressive control behavior. However, the additional measures would only compensate for and not alter the fundamental behavior of l_p formulations.

4. Conclusion

The main contribution of this paper has been to illustrate some of the consequences of using MPC with an l_p criterion. Our motivation for studying the l_p criterion was that for both l_1 and l_∞ criterion the resulting optimization can be formulated as a linear program. Linear programming formulations are desirable, because they are computationally less demanding than the standard quadratic programming formulations. Furthermore, theoretical issues such as stability are a straightforward extension of the results available for the quadratic criterion. However, performance issues raise questions concerning the suitability of the l_p criterion for MPC. Although possessing desirable theoretical and numerical properties, l_p formulations suffer many practical drawbacks. The main consequence of the l_p criterion is that it may yield either dead-beat or idle control performance. Both of these types of performance may be unsuitable for process control application.

While our arguments have been mostly qualitative, it is evident that the culprit is the non-smoothness of the objective function. The non-smoothness causes the stage cost functions to act as competing exact penalties for the constraints $u = 0$ and $y = 0$. For the scalar system, the behavior is simple to understand. Extending these results to higher dimension systems is more difficult and is currently unresolved.

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